Math/CS 466/666 **Programming Assignment 03**

Numerical Quadrature

Your work should be presented in the form of a typed report using clear and properly punctuated English. Where appropriate include full program listings and output. If you choose to work in a group of two, you must turn in independently prepared reports. Do not work in a group larger than two.

1a. Write a program that computes integrals using Simpson's method. Test your program with $n = 10$ to verify that you obtain

$$\int_1^5 \frac{1}{1+t^2}\,dt \approx 0.588037206324922.$$

1b. Let $g(t) = t^{p-1}e^{-t/2}$. Find t_0 such that $g(t_0) = \max\{\, g(t) : t \geq 0 \,\}$. Show for $M > t_0$ that

$$E_M = \int_M^\infty t^{p-1}e^{-t}\,dt = \int_M^\infty g(t)e^{-t/2}\,dt \leq 2M^{p-1}e^{-M}.$$

Find a value of M that ensures $E_M \leq 5 \times 10^{-16}$ for all $p \in [1,2]$.

1c. Let S_n be the approximation of $\Gamma(3/2) = \frac{1}{2}\sqrt{\pi}$ given by computing

$$\int_0^M t^{1/2}e^{-t}\,dt$$

using Simpson's method with n subdivisions. Compute S_n for $n = 2^k$ with $k = 4, \ldots, 10$. Plot a graph of $\log|\Gamma(3/2) - S_n|$ versus $\log h$. What is the slope of the graph? What should the slope have been?

1d. Make the substitution $t = u^5$ in the integral of part 1c. Approximate this transformed integral as R_n using Simpson's method with $n = 2^k$ subdivisions where $k = 5, \ldots, 8$. Plot a graph of $\log|\Gamma(3/2) - R_n|$ versus $\log h$. What is the slope of the graph?

1e. [Extra Credit and for Math/CS 666] Analyse the results of 1c and 1d in terms of the error estimates for Simpson's method. Note that

$$\frac{d^4}{dt^4}\left(t^{1/2}e^{-t}\right)$$

is unbounded on $(0, M)$.