Mandatory attendance unless special arrangement

Exam 1. on Friday Sept 24 **in person** at the usual classroom.

from last time

Then

$$Ax = \underbrace{\left[\, f(e_1) \,\middle|\, f(e_2) \,\middle|\, \cdots \,\middle|\, f(e_n) \,\right]}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x}$$

The inverse matrix is:

$$A^{-1} = \left[\, f^{-1}(e_1) \,\middle|\, f^{-1}(e_2) \,\middle|\, \cdots \,\middle|\, f^{-1}(e_n) \,\right]$$

How to find $f^{-1}(e_1)$, $f^{-1}(e_2)$, ..., and so on...

If $f(x) = y$ then by definition $f^{-1}(y) = x$.

- Vertical line test means only one solution for each equation $f(x) = e_k$ for each k.
- There needs to be at least one solution as well...

Solve $f(x) = e_1$,

→ $f(x) = Ax$

→ $Ax = e_1$

use augmented matrix...

$$\left[\, A \,\middle|\, e_1 \,\right]$$

row operation to construct the reduced row echelon form

Solve $f(x) = e_2$

$$\left[\, A \,\middle|\, e_2 \,\right].$$

and do the same thing again

really is the same, because the pivots depend on A and A is the same each time.

$$\left[\begin{array}{c|ccc} A & e_1 & e_2 & \cdots & e_n \end{array}\right]$$

(Only one solution and at least one solution).

means no free variables when solving for x

each row has a pivot

each row has a pivot means at least as many cols as rows

no free variables means no more cols than rows.

Conclusion A must be square if it's to have an (genuine) inverse...

EXAMPLE

32. $\begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix}$

Augmented Matrix

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 4 & -7 & 3 & 0 & 1 & 0 \\ -2 & 6 & -4 & 0 & 0 & 1 \end{array}\right] \quad (e_1\ e_2\ e_3)$$

Make reduced row echelon form...

$r_2 \leftarrow r_2 - 4r_1$

$r_3 \leftarrow r_3 + 2r_1$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 2 & -2 & 2 & 0 & 1 \end{array}\right]$$

$r_3 \leftarrow r_3 - 2r_2$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -4 & 1 & 0 \\ 0 & 0 & 0 & ? & -2 & ? \end{array}\right]$$

This system is inconsistent...so there is no inverse...for an example where there is an inverse please look at the 10am notes (which we also discussed in class)...

There are further examples in the textbook...