Course Map

Course Name: Math 330 Linear Algebra

Instructor Name: Eric Olson Date: Sun 02 Aug 2020 09:10:33 PM PDT

Program Outcomes Addressed: Algorithms, proofs, problem solving skills and increased mathematical maturity.

Official SLOs:

I. Find or identify a basis, use the Gram-Schmidt process to find an orthonormal basis and change a basis.

- II. Compute the dimension of a vector space, the rank of a matrix or the span of a collection of vectors.
- III. Compute eigenvalues and eigenvectors; determine whether a matrix is diagonalizable and if possible diagonalize it.

Additional Course Learning Outcomes:

- IV. Ability to translate practical problems in mathematics to vector and matrix notation.
- V. Use computer software and the formalism of linear algebra to calculate solutions.
- VI. Understand and clearly communicate theoretical proofs and logical arguments in writing.

Course Materials

Textbooks:

[1] Stephen Boyd and Lieven Vandenberghe, <u>Vectors, Matrices and Least Squares</u>, Cambridge University Press, 2018.

Resources:

- [2] Stephen Boyd and Lieven Vandenberghe, Vectors, Matrices and Least Squares--Julia Language Companion, Draft, 2019.
- [3] Lieven Vandenberghe, Additional Exercises, 2019, http://www.seas.ucla.edu/~vandenbe/133A/133A-exercises.pdf
- [4] Stephen Boyd and Lieven Vandenberghe, Additional Exercises, 2017, https://web.stanford.edu/class/ee103/103exercises.pdf
- [5] K.R. Matthews, Elementary Linear Algebra, 2013, http://www.numbertheory.org/book/mp103.pdf
- [6] Eric Olson, Lecture Notes on the Eigenvalue-Eigenvector Problem, 2020, in preparation.
- [7] Sheldon Axler, Linear Algebra Done Right, Spinger, 1997.

Module # and Title	Course Learning Outcome (CLOs)	Module Learning Outcomes (MLOs)	Assessments and Rubrics	Activities: Learner Interaction & Engagement	Instructional Materials
Module 1: Vectors	IV,V	 1.1 Remember vector notation and basic terminology. 1.2 Add vectors, take inner products and analyze how many arithmetic operations are required for each. 1.3 Understand vector operations in terms of real-world applications including location, color, time series, cash flow, images, audio and digital images. 1.4 Create vector models to represent story problems. 	Homework Pencil and paper computational to familiarize students with definitions and story problems from [1] to provide opportunities to create vector models of real-world problems. Exercises 1.5, 1.8, 1.11, 1.16, 1.17 in [1]. Assessments Quiz 1 and Final.	Computer Type in and try out the Julia examples in Chapter 1 of [2] interactively on your computer. Online Forum Post results and observations of Julia examples.	Read Chapter 1 of [1] and [2]. Technology Install Julia on computer; for how to do this see [2] and the syllabus.
Module 2 Linear Functions	IV,V	 2.1 Discuss notation, superposition and the definition of linear and affine functions. 2.2 Analyze first-order multi-variate Taylor approximations and regression models in terms linear and affine functions. 2.3 Evaluate the accuracy of linear models. 2.4 Create a linear model from real-world data to make predictions and optimize outcomes. 	Homework Exercises 2.1, 2.4, 2.10, 2.12 from [1]. Assessments Quiz 1 and Final.	Computer Try examples in Chapter 2 of [2].	Read Chapter 2 of [1].

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Module 3 Norm and Distance	VI	 3.1 Define norm, distance, standard deviation, angle between vectors and correlation. 3.2 Mathematically derive the triangle, Chebyshev and Cauchy-Schwarz inequalities. 3.3 Explain the meaning and relationship between the norm of a vector, RMS average and standard deviation. 3.4 Compute all the above and verify the mathematical inequalities by means of numerical examples. 	Homework Exercises 3.2, 3.3, 3.6, 3.7, 3.8, 3.19, 3.25 in [1]. Exercises 1.1, 1.2 in [3]. Exercises 3.1, 3.2 in [4]. Assessments Quiz 1 and Final.	Computer Try examples in Chapter 3 of [2] Online Forum Solutions and commentary about exercises 3.9, 3.15 in [1].	Read Chapter 3 of [1] and [2].
Module 4 Clustering	IV,V	 4.1 Real-world examples of clustering; explain the goal of clustering and what it means to be optimal. 4.2 Understand and apply the k-means algorithm. 4.3 Analyze data using k-means clustering and create recommendations based on the results. 	Homework Exercises 4.1abc in [1]. Assessments Quiz 2.	Computer Try examples in Chapter 4 of [2].	Read Chapter 4 of [1] and [2].
Module 5 Linear Indepen- dence	1	 5.1 Define span, independence, dimension, basis and orthonormal. 5.2 Analyze and apply the Gram-Schmidt algorithm and the modified Gram-Schmidt algorithm. 5.3 Evaluate the Gram-Schmidt algorithm in terms of computational complexity. 	Homework Exercises 5.2, 5.7 in [1] Exercises 5.1, 5.2 in [4]. Assessments Quiz 2 and Final.	Computer Try examples in Chapter 5 of [2]. Online Forum Early termination for gram_schmidt in 5.4 of [2].	Read Chapter 5 of [1] and [2].

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Module 6 Matrices	II	 6.1 Discuss matrix notation, terminology and the definition of matrix norm. 6.2 Add matrices, multiply them and analyze how many arithmetic operations are required for each. 6.3 Understand associativity, commutativity, identity and linear dependence of columns for matrices. 	Homework Exercises 6.2, 6.3, 6.6, 6.12, 6.13 in [1]. Assessments Quiz 2 and Final.	Computer Try examples in Chapter 6 of [2]. Online Forum Exercise 6.9, 6.10 in [1].	Read Chapter 6 of [1] and [2].
Module 7 Matrix Examples	VI	 7.1 Scaling, dilation, rotation, reflection and projection. 7.2 Vandermond, permutation and convolution matrices. 7.4 Directed graphs and networks. 	Homework Exercises 2.1, 2.5 in [3].	Computer Try examples in Chapter 7 of [2]. Online Forum Discussion of exercise 6.2 in [4] and 6.18 in [1].	Read Chapter 7 of [1] and [2].
Module 8 Linear Equations	VI	8.1 Examples of systems of linear equations.	Assessments None, skip if out of time.	Computer Try examples in Chapter 8 of [2].	Read Chapter 8 of [1] and [2].

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Module 9 Linear Dynamical Systems	IV,V	9.1 Definition of a Markov model.9.2. Population and epidemiological dynamics.	Homework Exercises 9.2, 9.3 in [1].	Computer Chapter 9 of [2]. Online Forum Model Washoe county COVID-19 using 9.3 of [1].	Read Chapter 9 of [1] and [2].
Module 10 Matrix Multiplication	I,VI	 10.1 Define matrix-matrix multiplication and understand the resulting algebraic properties. 10.2 Interpret matrix-matrix multiplication in terms of rows and in terms of columns. 10.3 Understand and apply QR factorization. 10.3 Analyze the computational complexity. 	Homework Exercises 10.3, 10.6, 10.11, 10.35 in [1]. Exercises 10.3, 10.7 in [4]. Assessments Quiz 3 and Final.	Computer Chapter 10 of [2]. Online Forum Discuss exercise 10.12 in [1] and post results for 10.1 in [4].	Read Chapter 10 of [1] and [2].
Module 11 Matrix Inverses	I,II,VI	 11.1 Define the left, right, pseudo and two-sided inverses. 11.2 Analyze the inverse of a product. 11.3 Apply the QR and LU factorization to solve linear equations and to compute the inverse. 11.4 Apply QR to under and over determined systems. 	Homework Exercises 4.1, 4.2, 4.8, 4.11, 7.4 in [3]. Assessments Quiz 3 and Final.	Computer Try examples in Chapter 11 of [2]. Online Forum Post results for 11.2 in [4].	Read Chapter 11 of [1] and [2].

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Module 12 Least Squares	I,II,IV,V	 12.1 Discuss the statement and significance of the least squares problem. 12.2 Apply the QR factorization to solve least squares. 12.3 Create a least squares solution to a story problem. 	Homework Exercises 12.3, 12.5, 12.8, 12.10 in [1] Assessments Final.	Computer Try examples in Chapter 12 of [2].	Read Chapter 12 of [1] and [2].
Module 13 Determinants	III,VI	13.1 Recursive definition of determinant.13.2 Determinant of a triangular matrix.13.3 Algebraic properties of the determinant.	Homework Exercises 4.2, 4.4ab, 4.5, 4.8 in [5] Assessments Final.	Online Forum Discuss using Julia to find the determinant of a matrix.	Read Chapter 4 of [5].
Module 14 Eigenvectors Eigenvalues	III	 14.1 Define eigenvectors, eigenvalues and the characteristic polynomial. 14.2 Remember and understand the significance of the spectral theorem for symmetric matrices. 14.3 Apply the characteristic polynomial to find eigenvectors and eigenvalues. 	Homework Exercises 6.1, 6.2 in [5] Exercises 1, 2, 3, 4 in [6]. Assessments Final.	Online Forum Discussion of the singular value decomposition.	Read Chapter 6 of [5] and [6].