Math 330 Homework 2 Version A

1. Let A, B, C and D be matrices defined by

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 4 \\ 5 & -1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \\ 2 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

and let n be a positive integer. Which of the following matrices are defined? Compute those matrices which are defined.

(i) $A + B$

(ii) AC

(iii) DB

(iv) DA

(v) $CD + A$

(vi) $DC + A$

(vii) $DC + \frac{1}{3}B$

(viii) A^n

(ix) B^n

2. Solve or show the following systems of equations are inconsistent by using the Gauss–Jordan algorithm to reduce the augmented matrix to reduced row-echelon form. Carefully write down each elementary row operation needed to find the reduced row-echelon form.

(i)

$$\begin{cases} x_1 + x_2 = 3 \\ x_1 - x_2 = 4 \end{cases}$$

(ii)

$$\begin{cases} 2x_1 - 6x_2 + 3x_3 - 2x_4 = -1 \\ \qquad -x_1 + 3x_2 - 2x_3 = 4 \\ 3x_1 - 9x_2 + 4x_3 - 4x_4 = 2 \end{cases}$$

(iii)

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 7 \\ 3x_1 + 2x_2 + x_3 + x_4 - 3x_5 = -2 \\ x_2 + 2x_3 + 2x_4 + 6x_5 = 23 \\ 5x_1 + 4x_2 + 3x_3 + 3x_4 - x_5 = 12 \end{cases}$$

3. Extra Credit: Work problems 3, 7 and 8 from Mathews pages 34–35. The answers are written in the text already. Make sure your work explains in terms of detailed calculations how to get the answers.