Math 330 Exam 1 Review

1. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 & 8 \\ -1 & 4 & 2 \\ -2 & 6 & 1 \end{bmatrix}$ and $v = \begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$.

(i) Compute $2A + B$.

(ii) Find Av.

(iii) Compute $\|v\|$.

2. Determine whether the following pairs of vectors are perpendicular.

(i) Is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ perpendicular to $\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$?

(ii) Is $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$ perpendicular to $\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$?

3. True or false with a counterexample if false and a reason if true.

(i) If A is invertible then A^{-1} is invertible.

(ii) If A is invertible then A^2 is invertible.

4. Let $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(i) Find Av.

(ii) Find A^2v.

(iii) Find A^3v.

5. Write the system of linear equations given by

$$\begin{cases} x_1 + 2x_2 = 4 \\ 2x_1 - x_2 = 3 \end{cases}$$

in the matrix form $Ax = b$. What is A? What is b?

6. Let $B = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$. Find B^{-1}.

7. Given the augmented matrix

$$\left[\begin{array}{ccc|c} 4 & 6 & 8 & 1 \\ 2 & 9 & -2 & 0 \\ 0 & 4 & 1 & 0 \end{array}\right]$$

the row operation $r_2 - (1/2)r_1 \to r_2$ yields the augmented matrix

$$\left[\begin{array}{ccc|c} 4 & 6 & 8 & 1 \\ 0 & 6 & -6 & -1/2 \\ 0 & 4 & 1 & 0 \end{array}\right].$$

(i) Find the elementary matrix E corresponding to this row operation.

(ii) What is E^{-1}?

8. Let

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 3 & 3 \\ 2 & 2 & 2 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}.$$

Solve the system $Ax = b$. Find x.

9. Is it true that

$$\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ b & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

for all values of a, b and c? If so explain why. If not find values of a, b and c for which there is inequality.

10. Let

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}.$$

Factor A into LDU where L is lower triangular, U is upper triangular with ones on the diagonal and D is diagonal.

11. Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 2 & 4 & 8 & 16 & 32 & 66 \end{bmatrix}$$

Find the reduced row echelon form R of A.

12. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 4 \\ 4 \\ 8 \end{bmatrix}$$

The nullspace of A is

$$\mathcal{N}(A) = \left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} c : c \in \mathbf{R} \right\}.$$

One solution to $Ax = b$ is

$$x = \begin{bmatrix} -1 \\ -10 \\ 5 \end{bmatrix}.$$

Find all solutions.

13. Let

$$A = \begin{bmatrix} 1 & 2 & 5 & 6 & 2 \\ 2 & 4 & 4 & 2 & 8 \\ 6 & 12 & 1 & 1 & -2 \\ 4 & 8 & -3 & -1 & -10 \end{bmatrix}.$$

The reduced row echelon form of A is

$$R = \begin{bmatrix} 1 & 2 & 0 & 0 & -1/2 \\ 0 & 0 & 1 & 0 & 7/2 \\ 0 & 0 & 0 & 1 & -5/2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(i) Find the rank of A.

(ii) Find a basis for the nullspace $\mathcal{N}(A)$. What is the dimension $\dim \mathcal{N}(A)$?

(iii) Find a basis for the column space $\mathcal{C}(A)$. What is the dimension $\dim \mathcal{C}(A)$?

14. Answer the following questions about linear independence, span and basis. Give a reason each of your answers.

(i) Are the vectors

$$\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

linearly independent?

(ii) Do the vectors $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ span the same space as $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$?

(iii) Do the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

form a basis for $\mathbf{R}^4$?

15. True or false with a counterexample if false and a reason if true.

(i) If A is any matrix, then $\mathcal{C}(A) = \mathcal{C}(A^T)$.

(ii) If A is any $n \times m$ matrix, then $\dim \mathcal{C}(A) = \dim \mathcal{C}(A^T)$.

(iii) If P is a permutation matrix, then $P^{-1} = P^T$.

(iv) If A is an $n \times n$ invertible matrix, then $\mathcal{C}(A) = \mathcal{C}(A^{-1})$.

16. Let

$$A = \begin{bmatrix} 1 & -1 & -2 \\ 2 & 1 & 2 \\ 4 & 1 & 8 \end{bmatrix}.$$

Find a lower triangular matrix L and an upper triangular U such that $LU = A$.

17. Let $B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. Find B^{-1}.

18. Let $A = LU$ where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

Solve the system

$$Ax = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Find x.