Math 285 Midterm Version A

1. Consider the initial value problem

$$\frac{dx}{dt} = f(x,t) \qquad \text{with} \qquad x(t_0) = x_0.$$

State the existence and uniqueness theorem which shows this ordinary differential equation has a unique solution on some open interval I containing t_0.

2. Solve the initial value problem $\dot{x} + 3x = e^{-3t}$ with $x(0) = 7$.

3. Draw a phase diagram for the autonoumous first-order ordinary differential equation $\dot{x} = x^3 - 5x$ on the line below. Label the stationary points with a cross $\times$ and draw arrows on the line indicating the direction in which $x(t)$ is changing.

4. Show that the ordinary differential equation

$$(y \cos x + 2xe^y) + (\sin x + x^2 e^y + 2)y' = 0$$

is exact and find the general solution.

5. Find the unique solution to $\dfrac{dy}{dx} = \dfrac{2x}{1+2y}$ with $y(0) = 1$.

6. Find the general solution to the differential equation

$$x^2 y' = y^2 + 2xy.$$

7. Find the general solution to the second order initial value problem $y'' + 5y' + 6y = 0$.

8. Find the unique solution to the second order initial value problem

$$y'' - 4y = 0 \qquad \text{with} \qquad y(0) = 1 \quad \text{and} \quad y'(0) = 0.$$

9. Consider the differential equation $\ddot{x} - 3\dot{x} - 4x = 2\sin t$.

(i) Find a particular solution for this differential equation.

(ii) Find the general solution to this differential equation.

(iii) Find the unique solution such that $x(0) = 1$ and $\dot{x}(0) = 4$.