

This section consists of a list of problems. The first 32 problems can be solved by the methods of the previous sections. They are presented so that the reader may have some practice in identifying the method or methods applicable to a given equation. At the end of the list are a number of problems suggesting specialized techniques that are useful for certain types of equations. In particular, Problems 35 and 36 deal with Riccati (1676–1754) equations.

1. $\frac{dy}{dx} = \frac{x^3 - 2y}{x}$
2. $(x + y) dx - (x - y) dy = 0$
3. $\frac{dy}{dx} = \frac{2x + y}{3 + 3y^2 - x}$, $y(0) = 0$
4. $(x + e^y) dy - dx = 0$
5. $\frac{dy}{dx} = -\frac{2xy + y^2 + 1}{x^2 + 2xy}$
6. $x \frac{dy}{dx} + xy = 1 - y$, $y(1) = 0$
7. $\frac{dy}{dx} = \frac{x}{x^2y + y^3}$ Hint: Let $u = x^2$.
8. $x \frac{dy}{dx} + 2y = \frac{\sin x}{x}$, $y(2) = 1$
9. $\frac{dy}{dx} = -\frac{2xy + 1}{x^2 + 2y}$
10. $(3y^2 + 2xy) dx - (2xy + x^2) dy = 0$
11. $(x^2 + y) dx + (x + e^y) dy = 0$
12. $\frac{dy}{dx} + y = \frac{1}{1 + e^x}$
13. $x dy - y dx = (xy)^{1/2} dx$
14. $(x + y) dx + (x + 2y) dy = 0$, $y(2) = 3$
15. $(e^x + 1) \frac{dy}{dx} = y - ye^x$
16. $\frac{dy}{dx} = \frac{x^2 + y^2}{x^2}$
17. $\frac{dy}{dx} = e^{2x} + 3y$
18. $(2y + 3x) dx = -x dy$
19. $x dy - y dx = 2x^2y^2 dy$, $y(1) = -2$
20. $y' = e^{x+y}$
21. $xy' = y + xe^{y/x}$
22. $\frac{dy}{dx} = \frac{x^2 - 1}{y^2 + 1}$, $y(-1) = 1$
23. $xy' + y - y^2e^{2x} = 0$
24. $2 \sin y \cos x dx + \cos y \sin x dy = 0$
25. $\left(2 \frac{x}{y} - \frac{y}{x^2 + y^2}\right) dx + \left(\frac{x}{x^2 + y^2} - \frac{x^2}{y^2}\right) dy = 0$
26. $(2y + 1) dx + \left(\frac{x^2 - y}{x}\right) dy = 0$
27. $(\cos 2y - \sin x) dx - 2 \tan x \sin 2y dy = 0$
28. $\frac{dy}{dx} = \frac{3x^2 - 2y - y^3}{2x + 3xy^2}$
29. $\frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}$
30. $\frac{dy}{dx} = \frac{y^3}{1 - 2xy^2}$, $y(0) = 1$
31. $(x^2y + xy - y) dx + (x^2y - 2x^2) dy = 0$
32. $\frac{dy}{dx} = -\frac{3x^2y + y^2}{2x^3 + 3xy}$, $y(1) = -2$